International Journal of Scientific & Engineering Research, Volume 7, Issue 4, April-2016
ISSN 2229-5518

Modeling and Controller Design of a UAV With Minimum Control Surfaces

Sruthy M.Panicker, Laila Beebi M., Y.Johnson

Abstract— The UAV is an acronym for Unmanned Aerial Vehicle, which is an aircraft with no pilot on board.UAVs can be remote controlled aircraft (flown by a pilot at a ground control station) or can fly autonomously based on pre-programmed flight plans or more complex dynamic automation systems. Generally UAVs have three control surfaces namely elevator, aileron and rudder for the control of pitch, roll and yaw movements. But the UAV used here has only two independently driven elevon surfaces. Though this type of design provides mechanical simplicity, unmanned aerial vehicles with only two elevon surfaces present interesting challenges in dynamics modeling. By using proportional-integral-derivative (PID) controllers and H-infinity controller,the flight performance of the system was analysed and obtained better results by using H-infinity controller compared to PID controllers.

Index Terms— Aileron, elevator, elevon, nonminimum phase system,plank,rudder,unmanned aerial vehicles.

1 Introduction

An unmanned aerial vehicle (UAV) is commonly known as a drone. It is also called by several other names.Unmanned aerial vehicle is an aircraft without a human pilot abroad. The flight of UAVs may be controlled either autonomously by onboard computers or by the remote control of a pilot on the ground in another vehicle[1]. Unmanned aircraft is classified into two types:

1) Autonomous aircraft
2) Remotely piloted aircraft

Fig.1.P15035 UAV, having only two independent elevon control surfaces.

Unmanned aerial vehicle is also known as UAS.The term unmanned aerial system(UAS) was adopted by the United States Department of Defence due to its applications in the field of military works.Unmanned aerial system includes elements such as the unmanned aircraft, ground control stations, data links and other related support equipment[2].Generally unmanned aerial vehicles have elevators, aileron and rudders as three control surfaces. The UAV used in this work have no rudder or elevators. Hence its roll, pitch and yaw movements are controlled by two independently driven elevon surfaces. This type of unmanned aerial vehicle is colloquially called a plank.

With the propellor mounted in a tractor configuration but having no rudder or elevators, its attitude control is done by two independently driven elevon surfaces, as shown in Fig. 1. It is very rugged, easy to maintain, of compact construction, and have better flight behaviour and wide airspeed range. It is because of these reasons, a similar configuration was adopted for the Dragon Eye, a small UAV for use by the U.S. Marine Corps[7].However, its two elevon (combining elevator and aileron) control surfaces result in an underactuated configuration and a significant coupling between roll and yaw, presenting a challenge for controller design[3].

2 Modelling Of Uav

According to Newton's second law of motion for each of the six degrees of freedom,

Mass * acceleration = disturbing force

For the rotary motion ,the mass and acceleration become moment of inertia and angular acceleration respectively.Also in this case the disturbing force become the disturbing moment or torque[4].

2.1 The Components of Inertial Acceleration

Consider the motion of a body referred to an orthogonal reference axis set oxyz[6].Let u, v, w be the components of velocity and X, Y, Z represents the components of force. Similarly p, q, r represents the components of angular velocity and L,M,N represents the moment components along x, y and z directions respectively[4].

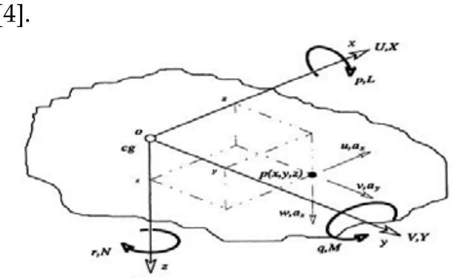

Fig.2.Motion referred to generalised body axis.

- *Sruthy M.Panicker is currently pursuing masters degree program in indusrial instrumentation and control engineering in TKMCE,Kollam*
- *Laila Beebi M. is Professor in industrial instrumentation and control engineering in TKMCE, Kollam.*
- *Y.Johnson is currently working as associate professor in UKFCET,Kollam*

The velocity components at p(x, y, z) are given by,

$$\begin{aligned} u &= \dot{x} - ry + qz \\ v &= \dot{y} - pz + rx \\ w &= \dot{z} - qx + py \end{aligned} \qquad (1)$$

Since the aircraft is assumed to be rigid,

$$\dot{x} = \dot{y} = \dot{z} = 0$$

Then,

$$\begin{aligned} u &= qz - ry \\ v &= rx - pz \\ w &= py - qx \end{aligned} \qquad (2)$$

Corresponding components of acceleration are given by,

$$\begin{aligned} a_x &= \dot{u} - rv + qw \\ a_v &= \dot{v} - pw + ru \\ a_z &= \dot{w} - qu + pv \end{aligned} \qquad (3)$$

The inertial velocity components are given by,

$$\begin{aligned} u' &= U + u = U - ry + qz \\ v' &= V + v = V - pz + rx \\ w' &= W + w = W - qx + py \end{aligned} \qquad (4)$$

Similarly, the inertial acceleration components are

$$\begin{aligned} a'_x &= \dot{U} - rV + qW - x(q^2 + r^2) + y(pq - \dot{r}) + z(pr + \dot{q}) \\ a'_v &= \dot{V} - pW + rU + x(pq + \dot{r}) - y(p^2 + r^2) + z(qr - \dot{p}) \\ a'_z &= \dot{W} - qU + pV + x(pr - \dot{q}) + y(qr + \dot{p}) - z(p^2 + q^2) \end{aligned} \qquad (5)$$

2.2 Generalised Force and Moment Equations

The total force components acting on the body are given by,

$$\begin{aligned} \sum \delta m a_x{}' &= X \\ \sum \delta m a_y{}' &= Y \\ \sum \delta m a_z{}' &= Z \end{aligned} \qquad (6)$$

Since the origin coincides with the cg,

$$\sum \delta mx = \sum \delta my = \sum \delta mz = 0$$

Thus the resultant components of total force acting on the rigid body were given by,

$$\begin{aligned} m(\dot{U} - rV + qW) &= X \\ m(\dot{V} - pW + rU) &= Y \\ m(\dot{W} - qU + pV) &= Z \end{aligned} \qquad (7)$$

Similarly,

$$\begin{aligned} \sum \delta m\,(ya'_z - za'_y) &= L \\ \sum \delta m\,(za'_x - xa'_z) &= M \\ \sum \delta m\,(xa'_y - ya'_x &= N \end{aligned} \qquad (8)$$

Thus the moment equation becomes,

$$\begin{aligned} I_x\dot{p} - (I_v - I_z)qr - I_{xz}(pq + \dot{r}) &= L \\ I_v\dot{q} + (I_x - I_z)pr + I_{xz}(x^2 - x^2) &= M \\ I_z\dot{r} - (I_x - I_v)pq + I_{xz}(qr - \dot{p}) &= N \end{aligned} \qquad (9)$$

2.3 Disturbance Forces and Moments

Assume that the disturbing forces and moments are dueto aerodynamic effects, gravitational effects, movement of aerodynamic controls, power effects and atmospheric disturbances. Then,

$$\begin{aligned} m(\dot{V} - pW + rU) &= Y_a + Y_g + Y_c + Y_p + Y_d \\ I_z\dot{r} - (I_x - I_v)qr - I_{xz}(pq + \dot{r}) &= L_a + L_g + L_c + L_p + L_d \\ I_z\dot{r} - (I_x - I_v)pq + I_{xz}(qr - \dot{p}) &= N_a + N_g + N_c + N_p + N_d \end{aligned} \qquad (10)$$

2.4 Linearised Equations of Motion

Initially, the aeroplane is assumed to be flying with zero roll, sideslip and yaw angles. Also,

$$X_d = Y_d = Z_d = L_d = M_d = N_d = 0$$

Then,

$$\begin{aligned} m(\dot{v} - pW_e + rU_e) &= Y_a + Y_g + Y_c + Y_p \\ I_x\dot{p} - I_{xz}\dot{r} &= L_a + L_g + L_c + L_p \\ I_z\dot{r} - I_{xz}\dot{p} &= N_a + N_g + N_c + N_p \end{aligned} \qquad (11)$$

2.5 Gravitational Terms

Since the origin of the aeroplane body axes is coincident with the cg there is no weight moment about any of the axes, therefore

$$L_g = M_g = N_g = 0$$

Thus the gravitational force components in the small perturbation equations of motion are given by,

$$\begin{aligned} X_g &= -mg\sin\theta_e - mg\cos\theta_e \\ Y_g &= mg\varphi\sin\theta_e + mg\emptyset\cos\theta_e \\ Z_g &= mg\cos\theta_e - mg\theta\sin\theta_e \end{aligned} \qquad (12)$$

2.6 Aerodynamic Terms

The aerodynamic term in the axial force equation can be expressed as,

$$Y_a = Y_{ae} + \mathring{Y}_u u + \mathring{Y}_v v + \mathring{Y}_w w + \mathring{Y}_p p + \mathring{Y}_q q + \mathring{Y}_r r + \mathring{Y}_{\dot{w}} \dot{w} \qquad (13)$$

The aerodynamic term in the rolling moment and yawing moment are given by,

$$\begin{aligned} L_a &= L_{ae} + \mathring{L}_u u + \mathring{L}_v v + \mathring{L}_w w + \mathring{L}_p p + \mathring{L}_q q + \mathring{L}_r r + \mathring{L}_{\dot{w}} \dot{w} \\ N_a &= N_{ae} + \mathring{N}_u u + \mathring{N}_v v + \mathring{N}_w w + \mathring{N}_p p + \mathring{N}_q q + \mathring{N}_r r + \mathring{N}_{\dot{w}} \dot{w} \end{aligned} \qquad (14)$$

The aerodynamic terms in the remaining equations of motion can be expressed in a similar way.

2.7 Aerodynamic Control Terms and Power Terms

The rolling moment and yawing moment due to aerodynamic controls can be expressed as,

$$L_c = \mathring{L}_\xi + \mathring{L}_\eta + \mathring{L}_\zeta$$
$$N_c = \mathring{N}_\xi \xi + \mathring{N}_\eta \eta + \mathring{N}_\zeta \zeta \qquad (15)$$

The normal force due to thrust can be expressed as,

$$Y_p = \mathring{Y}_\tau \tau$$

2.8 Lateral Equations of Motion

The aerodynamic coupling derivatives are negligibly small and hence,

$$\mathring{Y}_u = \mathring{Y}_{\dot{w}} = \mathring{Y}_w = \mathring{Y}_q = \mathring{L}_u = \mathring{L}_{\dot{w}} = \mathring{L}_w = \mathring{L}_q = \mathring{N}_u = \mathring{N}_{\dot{w}} = \mathring{N}_w = \mathring{N}_p = 0$$
$$\mathring{Y}_\eta = \mathring{Y}_\tau = \mathring{L}_\eta = \mathring{L}_\tau = \mathring{N}_\eta = \mathring{N}_\tau = 0 \qquad (17)$$

Thus the lateral directional equations of motion are,

$$m\dot{v} - \mathring{Y}_v v - p\mathring{Y}_p - (\mathring{Y}_r - mU_e)r - mg\phi = \mathring{Y}_\xi + \mathring{Y}_\zeta = 0$$
$$-\mathring{L}_v v + I_x\dot{p} - \mathring{L}_p p - I_{xz}\dot{r} - \mathring{L}_r r = \mathring{L}_\xi \xi = \mathring{L}_\zeta \zeta \qquad (18)$$

The state space equation of Linear Time Invariant (LTI) system is

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (19)$$

Then the lateral state equation can be written in the form

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} y_v & y_p & y_r & y_\phi \\ l_v & l_p & l_r & l_\phi \\ n_v & n_p & n_r & n_\phi \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} y_\xi & y_\zeta \\ l_\xi & l_\zeta \\ n_\xi & n_\zeta \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \zeta \end{bmatrix}$$

3 Elevon Controlled UAV

Conventional aircraft controls yaw, pitch and roll movements using rudder, elevator and aileron control surfaces[5]. Our UAV has only two elevon surfaces. The two elevons must control roll, pitch and indirectly yaw as there is no rudder. Let δ_L and δ_R be the left and right elevon angles, respectively and q, p, r be the rate vector of pitch, roll and yaw[7]. Then the model of elevon controlled UAV is given by,

$$\begin{bmatrix} q \\ p \\ r \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \\ G_{31} & G_{32} \end{bmatrix} \begin{bmatrix} \delta_L \\ \delta_R \end{bmatrix} \qquad (20)$$

The aircraft is symmetrical about the vertical plane. So the deflection of left and right elevons should be the same sign for pitch movement and opposite for roll and subsequently yaw. Thus,

$$G_{11} = G_{12}$$
$$G_{21} = -G_{22} \qquad (21)$$
$$G_{31} = -G_{32}$$

Then,

$$\begin{bmatrix} q \\ p \\ r \end{bmatrix} = \begin{bmatrix} G_q & 0 \\ 0 & G_p \\ 0 & G_r \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_D \end{bmatrix} \qquad (22)$$

where,

$$G_{11} = G_{12}$$
$$G_{21} = -G_{22} \qquad (23)$$
$$G_{31} = -G_{32}$$

Also δ_A and δ_D are the average and difference of two elevon deflections respectively.

4 PID Controller

A proportional–integral–derivative controller (PID controller) is a control loop feedback mechanism (controller) commonly used in industrial control systems.A PID controller continuously calculates an error value as the difference between a desired setpoint and a measured process variable.

By tuning the three parameters of the model,a PID controller can deal with specific process requirements. The response of the controller can be described in terms of its responsiveness to an error, the degree to which the system overshoots a setpoint, and the degree of any system oscillation.

Fig.3 shows the MATLAB SIMULINK block diagram of a UAV showing response of pitch angle to elevon with PID controller.

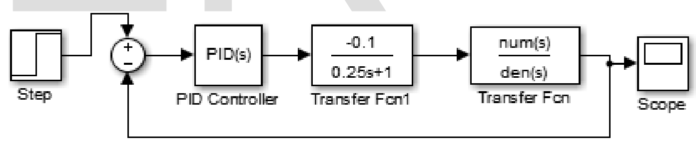

Fig.3.Block diagram of a UAV showing response of pitch angle to elevon

Fig.4 shows the MATLAB SIMULINK block diagram of a UAV showing response of roll angle to elevon with PID controller.

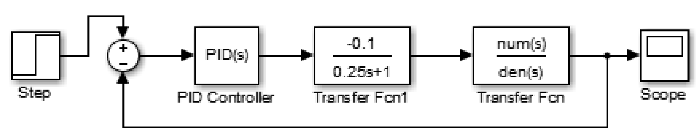

Fig.4.Block diagram of a UAV showing response of roll angle to elevon

Fig.5 shows the MATLAB SIMULINK block diagram of a UAV showing response of yaw angle to elevon with PID controller.

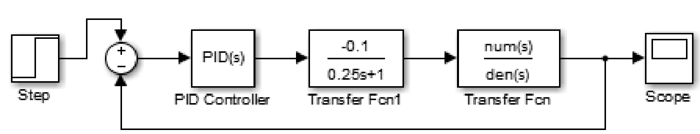

Fig.5.Block diagram of a UAV showing response of yaw angle to elevon

An actuator block of first order transfer function is also incorporated along with the plant transfer function.The PID controllers are tuned using optimization techniques.They have achieved the required control.While using PID controllers settling times are of the order of hundreds of seconds. But the required settling time is in the order of a few seconds. So we have to switch over to another controller capable of controlling each state within seconds.

5 H-INFINITY CONTROLLER

Under perturbed condition the conventional controllers like PID may become a failure in proper control. To maintain stability in real perturbed conditions, the robust H-infinity controller is taken for the given stability analysis. The general block diagram of H-infinity controller is shown in fig.6.

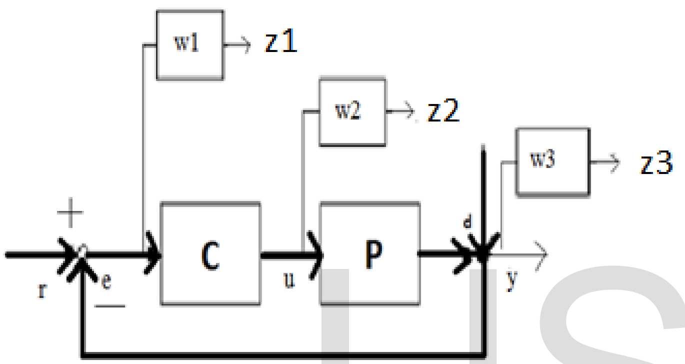

Fig.6.H-infinity controller and its weights along with the plant

Three weights namely W1, W2 and W3 are used to tune for achieving performance and stability. For achieving good disturbance rejection, the weight W1 should be properly selected. For stability margin,W3 should be tuned. W1 should be small inside the desired control bandwidth. The weight labelled W2 is known as control weight and is taken as an empty weight in this application. W1 is known as performance weight and W3 is known as stability weight. W1, W2 and W3 penalize the error, control and output signals respectively.

The necessary condition to be satisfied for the application of H-infinity controller is that the infinity norm of the product of weights assigned and the sensitivity(S)/complementary sensitivity (T) should be less than or equal to one. The SV (Singular Value) plots in fig.14,fig.16,fig.18 verified the application eligibility of H-infinity controllers in the present problem.

6 RESULTS

6.1 Open Loop Responses

The open loop responses of pitch angle,roll angle and yaw angle are shown in fig.7,fig.8,fig.9 respectively.On analysing the responses,it is clear that we must go for a suitable controller for obtaining the required performances.

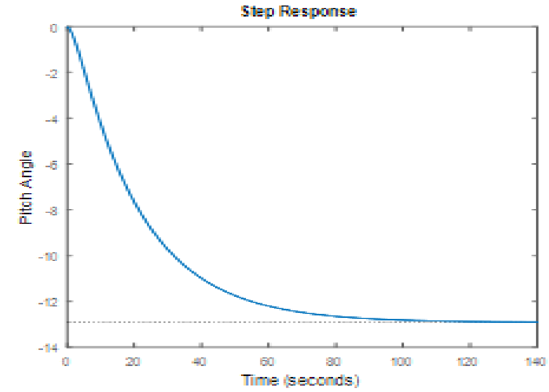

Fig.7.Response of pitch angle to elevon

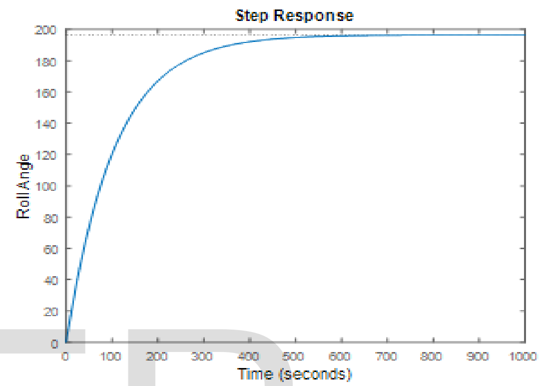

Fig.8.Response of roll angle to elevon

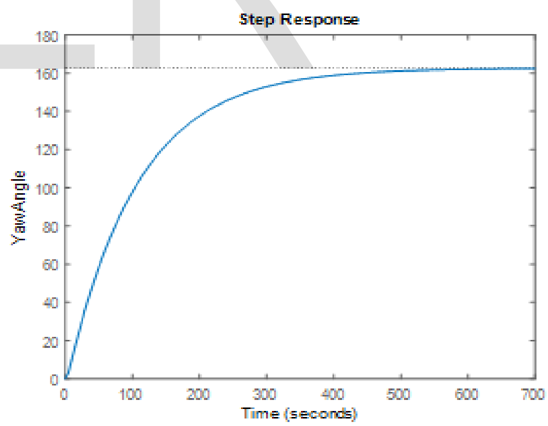

Fig.9.Response of yaw angle to elevon

6.2 Responses with PID controller

The responses of pitch angle,roll angle and yaw angle with PID controller are shown in fig.10,fig.11,fig.12 respectively.On analysing the responses,it is clear that their settling times does not meet the required specifications and there is a nonminimum phase in the yaw angle response.So we must go for another controller for obtaining the required performances.

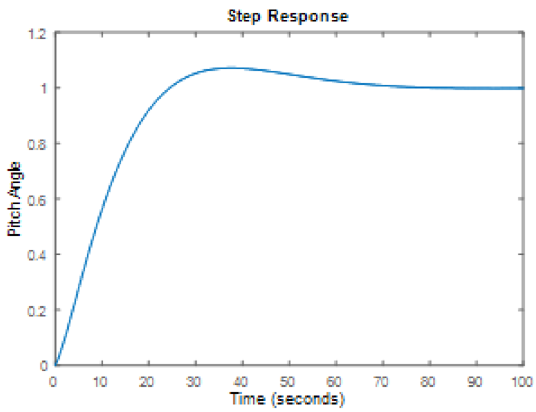

Fig.10.Response of pitch angle to elevon

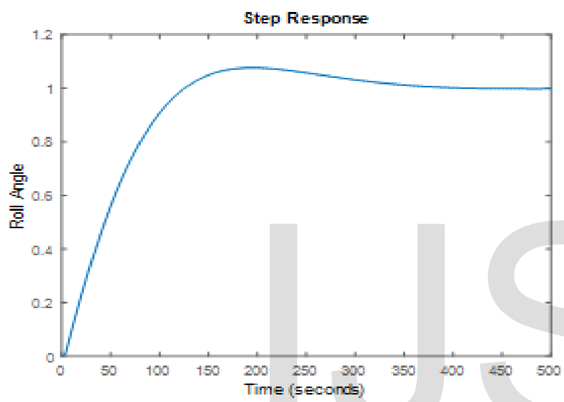

Fig.11.Response of roll angle to elevon

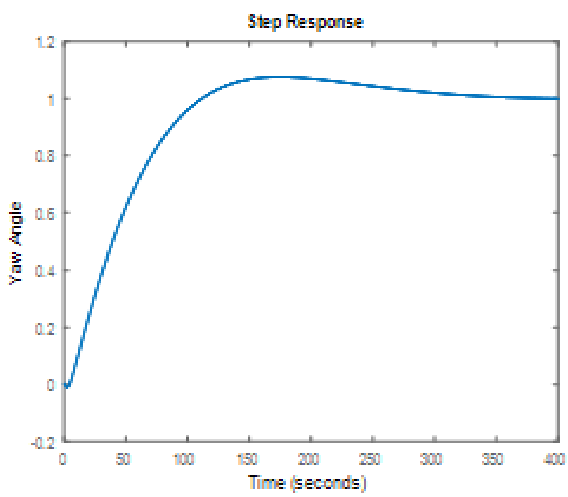

Fig.12.Response of yaw angle to elevon

6.3 Responses with H-infinity controller

The responses of pitch angle,roll angle and yaw angle with respect to H-infinity controller are shown in fig.13,fig.15,fig.17 and fig.14,fig.16 and fig.18 shows the corresponding SV plots respectively.On analysing the responses,it is clear that their settling times are of the order of few seconds and the nonminimum phase in the yaw angle response is eliminated.

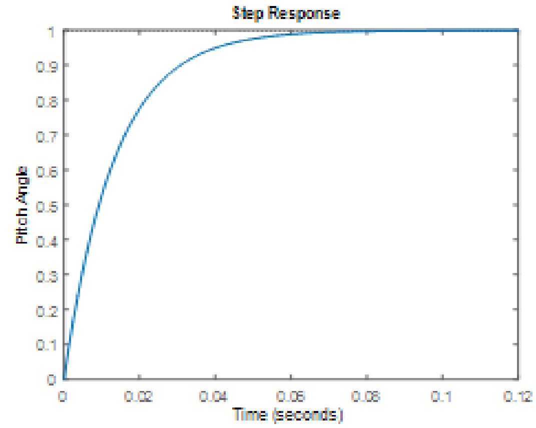

Fig.13.Response of pitch angle to elevon

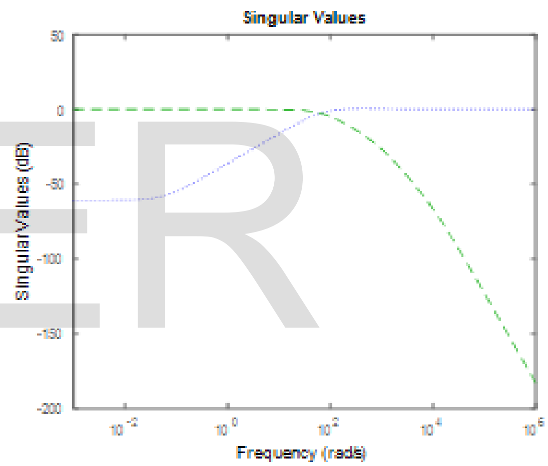

Fig.14.SV plot

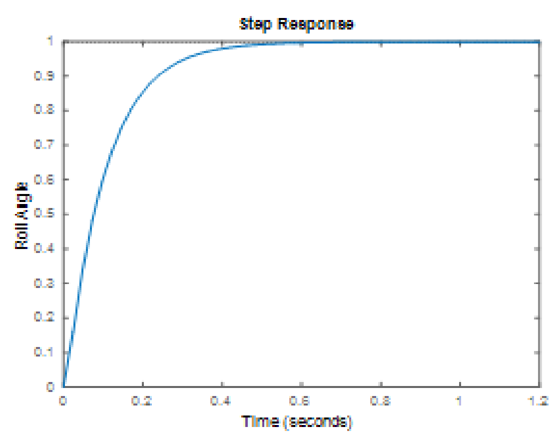

Fig.15.Response of roll angle to elevon

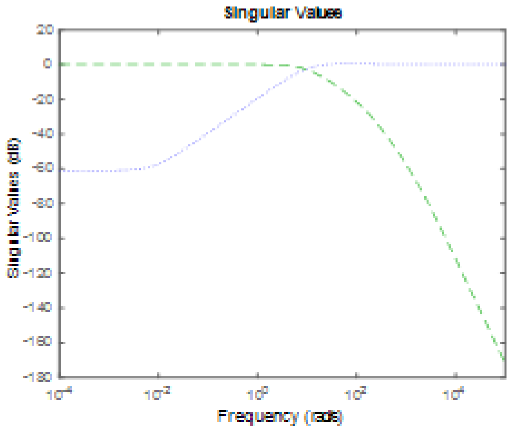

Fig.16.SV plot

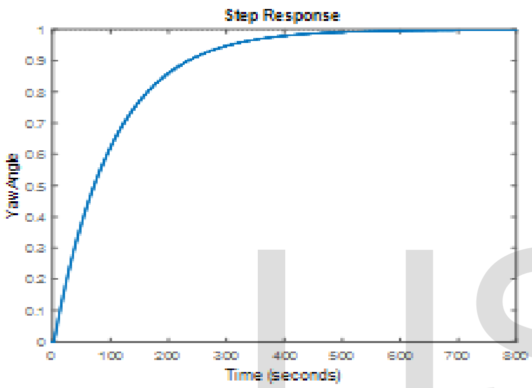

Fig.17.Response of yaw angle to elevon

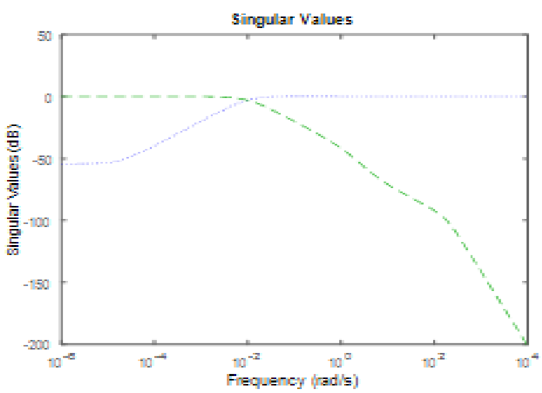

Fig.18.SV plot

7 Conclusion

This brief presented the modeling and test results for an unconventional UAV controlled using elevons without rudders or elevators. Through the fine tuning process, the PID controllers led to satisfactory flight performances with greater settling times and a nonminimum phase in the response of yaw angle. But by using H-infinity controller,settling times can be reduced compared to PID controllers and the problem of non-minimum phase system can be eliminated.

References

[1] Pratt,R.W.,"Flight Control Systems:Practical Issues in Design and Implementation," *IEEE Control Engineering Series*,IET,2000.

[2] Roskam J., "*Airplane Flight Dynamics and Automatic Flight Control*," Part-I,DAR Corporation,USA, 2003.

[3] Anderson J.D.,"*Introduction to Flight*," 2nd Edition,Mc Graw Hill Book Company

[4] Choon Seong, C., "*Generic UAV Modeling to Obtain its Aerodynamic and Control Derivatives*,"MS Thesis, Naval Postgraduate School, Monterey, CA. 2008.

[5] T. R.,Paul,Krogstad, and Gravdahl J.T., "*Modeling of UAV Formation Flight Using 3D Potential Field,Simulation,Modeling Practice and Theory*,"vol.16, Issue 9,pp.1453–1462,2008.

[6] M.V.Cook,"*Flight Dynamics*,"*Elsevier Ltd.*,pp.73-89.

[7] Ming Liu,Greg K.Egan,and Fendy Santoso,"Modeling,Autopilot Design,and Field Tuning of a UAV With Minimum Control Surfaces," *IEEE Trans.Control Systems Technology*,15 Jan 2015.